Option Price Estimations and Speculative Trading In Knowledge Society

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Derivatives market has known an enormous and continuous development from the late 1970s, thanks to the most celebrated Black-Scholes-Merton formula. The impact on global economy is also tremendous, but due to the high leverage of speculative option trading there is a perpetual danger of economic collapse. This paper gives a short description of knowledge society and proposes methods for option price estimation based on implied volatility, skewness and kurtosis. 'Free-lunch' is hardly achievable if one predicts the option price using the knowledgeable information from the market and there is almost impossible to speculate, rather than to hedge, when trading option.
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1 Introduction

The original 1973 Black-Scholes [1] and Merton [2] models and formulae for option pricing are probably the most important in financial theory and can be credited for a major turn-over in capital markets transactions since its appearance. Both [1] and [2] have witnessed improvements in relaxing some of the initial restrictive assumptions, like models of Bakshi, Cao and Chen [3], Corrado and Su [4] [5]. Still, the original one is the cornerstone and, comparing to the new models, more preferred for its simplicity by the majority of traders.

The objective of this paper is to prove that good estimations of option price can be achieved using the information provided by the options market. In order to make this proof we will use some implied variables in two formulae: implied volatility in [1], volatility, skewness and kurtosis in modified [5] formula of Negrea, Maillet and Jurczenko [6]. Implied variable or variables, in an option pricing formula, yield a theoretical value for the option price equal to the current market price of the option.

We would like to introduce the notion of autonomy of an option, meaning each option depend only on its underlying asset, but not on other options, even if they share the same underlying asset. Thus, implied variables are calculated separately for each individual option. There are two approaches in establishing the implied variables in this paper, and both are taking into consideration the fact that implied volatility, skewness and kurtosis are measures of risk for each option independently.

- the first approach (A1) is that the market retains the information and quickly and adaptively transforms it. Implied volatility, determined at a previous time, from the last observed transaction, can be introduced in [1] at the current time in order to determine a predicted option price. This approach is good when the variations of the stock price are small, but has an important role in proving, empirically, that an option on an underlying asset is dependent on this asset, but independent from the other options that share the same underlying. Using implied volatility, skewness and kurtosis in [6] we will use the previous three transactions to estimate the option price of an option to any given moment;
- the second approach (A2) takes into consideration that an option is valued based on transactions from a farther moment of its life than in first approach. Thus, the implied volatility is a result which, obtained by a non-linear minimization and introduced in [1], verifies in the best way (through a Euclidian distance) the traded prices of the derivative since its birth (or some given transaction) to the last transaction. In case of implied volatility, skewness and kurtosis we will use a non-linear minimization for three parameters through a Euclidian distance. We introduce, on this occasion, an original iterative method (IM) that finds a value for an implied variable in one parameter non-linear minimization.

Data used in research consists in more than 35 000 observations (Appendix 1) and is taken from the French CAC 40 Index from the 2nd of January 1997 to the 30th of December 1998, a period that covers the Asian financial crisis.

We provide lines of code source for some procedures implemented in Matlab and we also make

appeal to algorithms and functions already implemented in Matlab.
This paper is organized as follows: in section two we present a definition of knowledge society, formulae used in this research are displayed in the third section, *IM* and some other numerical methods, used in this paper to estimate option prices, are presented in section 4, in the fifth section we discuss the results obtained with different approaches and methods and the last section presents the most important conclusions.

2 Knowledge Society and Option Trading

For the last decades there have been many attempts to define information society and, more lately, some others for knowledge society. Based on the online version of Merriam-Webster Dictionary [7], a literally explanation of the expression knowledge society takes into account a third common notion: association. We regard knowledge as a condition of knowing something through experience and association, learnt and understood by reasoning on truth. Society is a voluntary association of individuals that shares common goals, beliefs, traditions, an interdependent system of biological units. Briefly, knowledge society is a large community of people that are dependent on each other and each of its members is equally important as one's work and experience is a strong contribution to achieving individual and common ends. Information is another key concept for knowledge society and, according to UNESCO [8] we cannot speak of a global society when information is impeded, manipulated and/or censured. More, information and communication technologies (ICT) are not means to an end but to create knowledge societies that: "are about capabilities to identify, produce, process, transform, disseminate and use information to build and apply knowledge for human development" [8] In a knowledge society we build our cooperation on trust, empathy and respect and we achieve them through a continuous collaboration between its individual members. This society is a democratic one, highly participative, not only in political processes, but more of a social signification. The cooperation is based on non-sum-zero games, and the attempt to default from one member leads to poorer individual and common results [9].
How do we look at option trading and what is its purpose in the context of knowledge society? Its original goal is to limit the risk of increase or decrease of the underlying asset price when the last one is subject to direct trading. Nowadays, the end for trading options is more speculative, either to discover the real price of the underlying asset [10] or to simply speculate in the options market, based on the information asymmetry or differences of opinion [11]. While "the productive or knowledge-increasing financial wager enlarges knowledge" speculative option trading might be a cry for help: "For aristocrats, workers and marginalized ethnic groups, gambling served the purpose of setting the culture of risk apart from the culture of control" [12]. But knowledge society is a large community that shelters everyone who is willing to cooperate for the benefit of all and where there is no room for exclusion. Still, speculative trading does affect not only the speculators but all the other members of the community, and if the monitoring and diminishing of such activities is a first solution [8] the major action should be taken by speculators themselves who are supposed to become more responsible.
In this paper we try to prove that speculation is hardly achievable on option trading, as the market itself gives enough information to estimate a fair price based on previous transactions of an option.

3 Option Pricing Methods

We have already pointed up that there are two approaches used in this paper to determine the implied variables needed for option pricing. These two approaches will be applied for each of the two models of option pricing, and each of the models has a specific formula.
We start by displaying the most consecrated Black-Scholes [1] formula and we also introduce some standard notations used in this paper:

- t : any given time or current time
- T: expiration date
- $T-t$: duration of the option
- C_t: call price at t time
- S_t: underlying price at t time
- N: cumulate distribution function
- K: exercise/strike price
- r: annualized risk-free interest rate, continuously compounded
- σ: historical volatility – a constant value

$$C_t = S_t N(d_1) - Ke^{-r(T-t)} N(d_2)$$

where $d_1 = \dfrac{\ln\left(\dfrac{S_t}{K}\right) + \left(r + \dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

and $d_2 = \dfrac{\ln\left(\dfrac{S_t}{K}\right) + \left(r - \dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$

We bring one amendment to formula (1) such as σ is not considered a fixed value, but an implied volatility and it is calculated using the two approaches already described:

- σ_{t-1} is the implied volatility, determined at a previous time *t*-1, from the last observed transaction. It is introduced in formula (1) at current time *t* in order to determine an estimated call option price CE^1_{t-1}.
- $\sigma_{x,t-1}$ is the implied volatility of an estimated price $CE^1_{x,t-1}$, obtained by a non-linear minimization. Introduced in formula (1), verifies in the best way (through a Euclidian distance) the traded prices of the option since the first chosen transaction at time *x* until the previous last transaction at time *t*-1.

The model of Corrado and Su [5] is practically an extent of Black-Scholes model [1] from the perspective of the existence of the first five statistical moments of the non-normal distribution of the underlying return. Using volatility, skewness and kurtosis, the call option price formula (2) presented in this paper is a variation of [5] and belongs to Negrea et al. [6]:

- $\gamma_1 = \dfrac{\mu_3}{\mu_2^{3/2}}$

  $C_{CS} = C^*_{BS} + \gamma_1(f)Q'_3 + \gamma_2(f)Q'_4$ and

  $\gamma_2 = \dfrac{\mu_4}{\mu_2^2}$, μ_i are the moments of normal distribution function *f*, *i* = 2,3,4
- $\omega = \dfrac{\gamma_1(f)}{3!}\sigma^3\tau^{3/2} + \dfrac{\gamma_2(f)}{4!}\sigma^4\tau^2$, ω is a constant value
- $d^* = (\sigma\sqrt{\tau})^{-1}\left[\ln\left(\dfrac{S_t}{Ke^{-r\tau}}\right) + \dfrac{1}{2}\sigma^2\tau - \ln(1+\omega)\right]$,

  d^* is the corrected value of moneyness
- $\varphi(x) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{x}{2}}$, $\varphi(x)$ is the density function
- $P_1(y) = 2\sigma\sqrt{\tau} - y$
- $P_2(y) = y^2 - 3y\sigma\sqrt{\tau} + 3\sigma^2 - 1$
- $Q'_3 = \left[3!(1+\omega)\right]^{-1} S_t\sigma\sqrt{\tau}P_1(d^*)\varphi(d^*)$
- $Q'_4 = \left[4!(1+\omega)\right]^{-1} S_t\sigma\sqrt{\tau}P_2(d^*)\varphi(d^*)$
- C^*_{BS} is the call option price from formula (1) with d^* instead of d_i, *i*=1,2
- C_{CS} is the option price of the model [5] modified in [6]

  $$C_{CS} = C^*_{BS} + \gamma_1(f)Q'_3 + \gamma_2(f)Q'_4 \qquad (2)$$

We also bring a few amendments to formula (2) and we take σ as an implied volatility σ^{cs}, γ_1 as an implied skewness s^{cs} and γ_2 as an implied kurtosis k^{cs}. The same two approaches, A1 and A2, are used to calculate the three implied values:

- $\sigma^{cs}_{t-3,t-1}, s^{cs}_{t-3,t-1}, k^{cs}_{t-3,t-1}$ are implied volatility, skewness and, respectively, kurtosis of the estimated call price CE^2_{t-3}, calculated using the previous three transactions, from moment of time *t*-3 to moment of time *t*-1.
- $\sigma^{cs}_{x,t-1}, s^{cs}_{x,t-1}, k^{cs}_{x,t-1}$ are implied volatility, skewness and, respectively, kurtosis of estimated call price $CE^2_{x,t-1}$, calculated from the first chosen transaction at time *x* to previous transaction at time *t*-1.

4 Numerical Methods

There are four types of implied variables that have to be calculated, using formulae (1) and (2) for both approaches A1 and A2:

- V_1: σ_{t-1}
- V_2: $\sigma_{x,t-1}$
- V_3: $\sigma^{cs}_{t-3,t-1}, s^{cs}_{t-3,t-1}, k^{cs}_{t-3,t-1}$
- V_4: $\sigma^{cs}_{x,t-1}, s^{cs}_{x,t-1}, k^{cs}_{x,t-1}$

We will present numerical methods that yield the values of variables for each of V_i, i = 1, 4 and we also make some comments on these methods.

4.1 Bisection versus Newton-Raphson Method in Calculating V_1

There are two main ways to solve a non-linear equation: bisection method or methods that use

derivatives of the equation's function. While bisection method is not as fast as methods that use derivatives of a function, it is more accurate, always converging to a result, if this result exists.
We provide Matlab implementation for both bisection method in Appendix B1 and Newton-Raphson (tangent) method in Appendix B2. For bisection method we use an iterative and also a recursive approach, and in both cases we obtain the same correct result. Two approaches of Newton-Raphson method lead to the same incorrect result, using, in the first case, a numeric derivative of function and, in the second case, the analytical Vega. We remind that Vega is the derivative of call option function (1) with respect to σ. Using Bailey's numerical method, that takes into consideration not only the first derivative, but also the second derivative of call option function with respect to σ, will only accelerate the behavior of tangent method, leading to an inaccurate result with fewer iterations.
Figure 1 illustrates the comparison between bisection and Newton-Raphson and how each method yields its result:

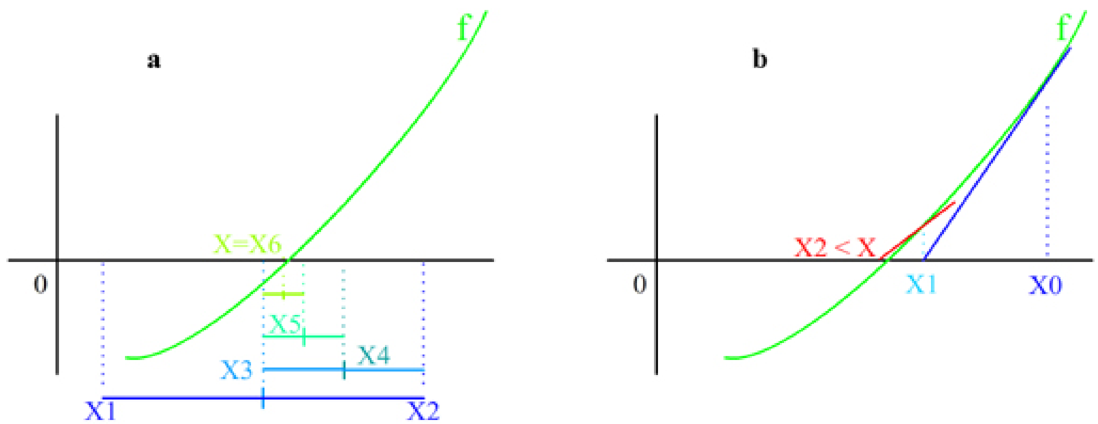

Fig. 1. Comparison between bisection (a) and Newton-Raphson (b)

As Figure 1.a shows, bisection method needs two initial guess solutions *X1* and *X2* and the interval determined (*X1*, *X2*) must contain the final solution *X*. In case of tangent method, see Figure 1.b, the method does not converge to a correct result *X*, but jumps the value of *X*, converging to a value close to zero, see Table 1. The lack of convergence might appear due to the fact that call option function does not have a regular shape as function ***f*** in Figure 1.
We conclude our investigations on methods that find a good solution for V1 by presenting and explaining some results obtained, for different transactions, with bisection and tangent method. Table 1 shows a comparative analysis of the two methods, for a precision 10^{-4} of σ_{t-1} :

Table 1. Comparison between bisection (M1) and tangent (M2) methods

C	K	S	r	T-t	**V₁:** σ_{t-1}		$f(\sigma_{t-1})$	
(currency)	(currency)	(currency)	(%)	(days)	M1	M2	M1	M2
130	2200	2283.02	3.41	84	0.1605	-0.0074	130.0009	-7.11E-37
165	2200	2286.91	3.41	271	0.0961	-0.0061	165.0005	-1.36E-34
43	2350	2265.99	3.41	83	0.1627	0.005	43.0011	5.30E-33
55.6	2350	2304.41	3.41	80	0.1588	0.0028	55.6019	2.77E-21
355	3550	3391.12	3.54	189	0.4072	0.0036	354.9985	2.03E-26
61	4450	2984.03	3.55	363	0.3052	0.0265	61.0013	1.47E-42

While tangent method yields results close to zero and even negative values, see first two lines of Table 1, bisection method always converges to a decent result. More, it takes around 20 iterations for bisection method to yield the result, while tangent method converges to an inaccurate value of σ after 100 iterations or so. For bisection method, verifying the values of σ_{t-1} and its corresponding variables (i.e. *K*, *S*, *r*, *T-t* in Table 1), with formula (1) we always obtain a value of $f(\sigma_{t-1})$ equal, for a precision of 10^{-2}, to observed price of the call option (i.e. *C* in Table 1). In case of Newton-Raphson method, introducing the resulted σ_{t-1} and its corresponding values in formula (1) we only obtain a value of $f(\sigma_{t-1})$ equal to zero, for a very high precision of 10^{-20}.

4.2 Iterative method (IM) for One Parameter Non-Linear Minimization in a Series of Data

We will use IM to calculate V_2, but we will also use IM in further researches to determine different values for initial solutions for variables of V_3 and V_4.

Let us introduce some notations:

- n - number of transactions: from the first or chosen one to the previous transaction
- C_i - real call option price in transaction *i,* a given value
- CE_i - estimated call option price in transaction *i,* a function: $CE_i = CE_i(\sigma)$

Let us take:

$$d\left(C,CE(\sigma)\right)=\sqrt[2]{\sum_{i=1}^{n}\left(C_i-CE_i(\sigma)\right)^2},\ d\left(C,CE(\sigma)\right) \text{ is minimum}$$

Let us define:

$$Q(\sigma)=\sum_{i=1}^{n}\left(C_i-CE_i(\sigma)\right)^2, \text{ hence } \sigma_{x,t-1} \text{ is that value of } \sigma \text{ which makes } Q(\sigma) \text{ minimum}$$

Let us read:

$$\sigma_1\ ,\sigma_5\ ,\ \varepsilon$$

and take

$$\sigma_3=\frac{\sigma_1+\sigma_5}{2}$$

where σ_1 and σ_5 are two initial values, the lower and upper bounds for the result, stored in a vector σ with 5 elements. ε is the level of accuracy, also called the tolerance or the precision.

Step 1

$$\sigma_2=\frac{\sigma_1+\sigma_3}{2}\ ,\sigma_4=\frac{\sigma_3+\sigma_5}{2}$$

Step 2

$$v_i=Q(\sigma_i)\ ,\ i=1,5$$

$$(a_1\ ,a_2\ ,\ a_3\ ,a_4\ ,\ a_5)=(v_1\ ,v_2\ ,\ v_3\ ,v_4\ ,v_5)$$

Step 3

Ascending sorting of the vector $a=(a_1\ ,a_2\ ,\ a_3\ ,a_4\ ,\ a_5)$

Step 4.1

Take $a_1,a_2\ ,a_3$ and determine their positions p_1,p_2,p_3 in vector $v=(v_1,v_2\ ,v_3\ ,v_4\ ,v_5)$, such as $p_1<p_2<p_3$

Step 4.2

IF NOT

($p=(1,2,3)\ or\ p=(2,3,4)\ or\ p=(3,4,5)$ **)**

stop!

The algorithm could not find a global minimum.

ELSE

IF $p_1=1$, **THEN** $\sigma_5=\sigma_3\ and\ \sigma_3=\sigma_2$

ELSE IF $p_1=2$ **THEN**

$\sigma_1=\sigma_2\ and\ \sigma_5=\sigma_4$

ELSE $\sigma_1=\sigma_3\ and\ \sigma_3=\sigma_4$

Step 5

IF $\left|\sigma_1-\sigma_5\right|\le\varepsilon$ **, THEN**

$\sigma_{x,t-1}=\sigma^*$, where $v(\sigma^*)=\min\left(v(\sigma)\right)$

STOP!

ELSE (i.e. $\left|\sigma_1-\sigma_5\right|>\varepsilon$ **)** ***GO TO Step 1.***

We would like to point up the importance of IM by comparing with an algorithm that uses the first derivative of the function that describes any of the observation in a series of data. IM has the same behavior as bisection method, which is a source of inspiration for IM. It requires two initial solutions, instead of only one for a method that utilizes a first derivative, and it is slower. But it does not depend on the initial solution to converge to a decent result, for a given precision. More, it does not require a proof for the continuity of the function which is derived, even if derivation is applied numerically and not analytically.

In case of methods with derivatives, non-linear minimization might yield a local minimum and they depend on how far is the initial solution from the final result. We will make a proof for the accuracy of IM (see Matlab implementation in Appendix B3), by comparing with Matlab function *lsqnonlin* [13], which may use one of the two algorithms: Levenberg [14], rediscovered by Marquardt (LM) [15], or trust region interior-reflective Newton algorithm (TR) [16]. When calculating V_2, the initial solution for methods implemented by *lsqnonlin* is not very important, even for values which are far from the final result, but it is important when calculating V_3 and

V_4. Table 2 present the proof that IM is consistent and accurate for a precision of 10^{-5}:

Table 2. Comparison between IM, Levenberg-Marquardt (LM) and trust region (TR)

No. of obs.	**C**	**K**	**S**	**r**	**T-t**	**V_2: $\sigma_{x,t-1}$**		
	(currency)	(currency)	(currency)	(%)	(days)	IM	LM	TR
1	195	2500	2608.54	3.32	217	-	-	-
2	195	2500	2610.05	3.32	217	0.123277	0.123277	0.123277
3	170	2500	2562.56	3.38	194	0.132041	0.132041	0.132041
4	172	2500	2567.17	3.38	194	0.135732	0.135733	0.135733
5	174	2500	2574.46	3.38	194	0.136986	0.136984	0.136984
6	174	2500	2573	3.38	194	0.138083	0.138080	0.138080

Non-linear minimization was applied to obtain V_2, using antecedent observations for each of implied volatilities from Table 2. The values of $\sigma_{x,t-1}$ are identical for LM and TR and almost identical for IM. This proves that IM yields good results and we would also like to prove that it does not depend on its initial guess solution. In Table 3 we calculate V_2 for different initial solutions (bounds) of $\sigma_{x,t-1}$, using the following notations: LB – lower bound, UB – upper bound and NI – number of iterations.

Table 3. V_2 with different LB and UB

No. of obs.	**LB = 0**	**UB = 1**	**LB = 0**	**UB = 5**	**LB = 0**	**UB = 10**	**LB = 0**	**UB = 100**
	V_2: $\sigma_{x,t-1}$	NI	**V_2: $\sigma_{x,t-1}$**	NI	**V_2: $\sigma_{x,t-1}$**	NI	**V_2: $\sigma_{x,t-1}$**	NI
2	0.12327576	**18**	0.12327671	**20**	0.12327671	**21**	0.12327731	**25**
3	0.13204193	**18**	0.13204098	**20**	0.13204098	**21**	0.13204217	**25**
4	0.13573456	**18**	0.13573170	**20**	0.13573170	**21**	0.13573170	**25**
5	0.13698578	**18**	0.13698578	**20**	0.13698578	**21**	0.13698339	**25**
6	0.13808060	**18**	0.13808250	**20**	0.13808250	**21**	0.13808012	**25**

It is obvious that bounds of IM do not affect consistently, for a precision of 10^{-5}, the resulted values of $\sigma_{x,t-1}$, see Table 3. There is also important to point up the fact that a larger interval for the initial solutions has little influence on the number or iterations that IM requires to yield the optimum result, see Figure 2. Thus, we can use IM without being concerned that the final solution might not be close to an initial solution, required by algorithms based on derivative of a function. This will be very helpful especially when calculating V_3 and V_4, which have a much larger range for their optimum solutions.

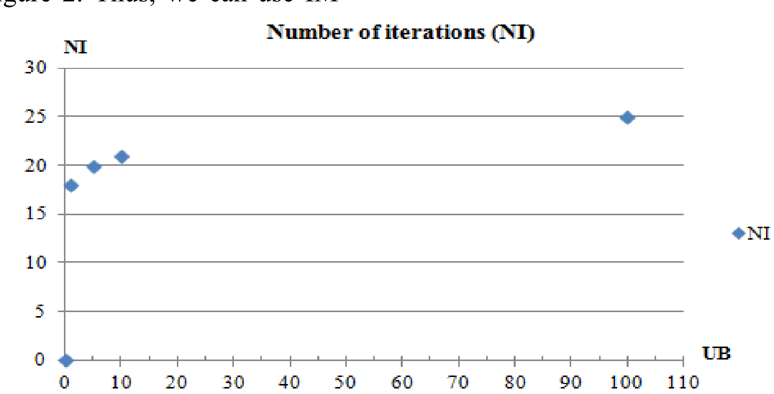

Fig. 2. Logarithmical dependency between NI and UB for IM for a precision of 10^{-5}

Figure 2 illustrates a logarithmical dependency between NI and UB, although we only used five observations. Still, this encourages us to believe, see also Table 3, that IM is not substantially affected by large intervals between the two initial guess solutions.

4.3 Methods to Determine V_3 and V_4

Values of V_3 and V_4 are calculated using formula (2) and both approaches A1 and A2. For both V_3 and V_4, the following steps are required in order to obtain initial solutions:

- Using *IM*, first step finds the implied volatility σ^{cs} with formula (1).
- Using *IM*, the implied volatility σ^{cs} from first step is introduced in formula (2) and we obtain the implied skewness s^{cs} when implied volatility σ^{cs} is fixed and kurtosis k^{cs} is neglected (i.e. $\gamma_2 = 0$). Thus, formula that is used to find s^{cs} becomes:
$$C_{CS} = C_{BS}^{*} + \gamma_1(f)Q_3^{'}$$
- Finally, using *IM*, implied volatility σ^{cs} and skewness s^{cs} from previous steps are introduced in formula (2) and implied kurtosis k^{cs} is found.

The initial values $\sigma^{cs}, s^{cs}, k^{cs}$ are used as guess solution in solving a system of non-linear equations in case of V_3 and in a non-linear minimization through a series of data in case of V_4. In order to find final solutions of V_3 and V_4 we will use Matlab functions: *fsolve* or *lsqnonlin* [13] for V_3 and *lsqnonlin* for V_4, which have implementations for LM and TR.

We must say that using IM we offer a good initial for LM in both approaches A1 and A2 for V_3 and V_4. Otherwise, LM has issues in calculating distribution function for complex values that it occasionally generates in its iterations. TR does not have this kind of problems, but it uses bounds that restrict the final results and it becomes an option when we want to obtain implied V_3 and V_4 that still keep their statistical significance (i.e. negative asymmetry and positive kurtosis).

We would like to underline the differences between results obtained with LM or TR, using or not using IM for initial solutions. Table 4 presents results of V_3, obtained from observations from Table 2, using TR and LM:

Table 4. Results for V_3 with LM and IM

No. of. obs.	*fsolve*: **LM** with **IM**			*lsqnonlin*: **LM** with **IM**			*lsqnonlin*: **LM**		
	$\sigma^{cs}_{t-3,t-1}$	$s^{cs}_{t-3,t-1}$	$k^{cs}_{t-3,t-1}$	$\sigma^{cs}_{t-3,t-1}$	$s^{cs}_{t-3,t-1}$	$k^{cs}_{t-3,t-1}$	$\sigma^{cs}_{t-3,t-1}$	$s^{cs}_{t-3,t-1}$	$k^{cs}_{t-3,t-1}$
3	0.0288	-167.0899	585.5107	0.0393	-34.8398	11.6626	0.0402	-30.7379	3.2882
4	0.0596	-16.9003	5.5834	0.0483	-28.1049	23.6895	0.0184	1.1396	8.1237
5	0.0479	-36.9400	35.8936	0.0453	-39.8313	40.5228	0.0432	-19.6885	-16.5300
6	0.0243	-214.4738	1247.0040	0.0482	-37.5249	38.4458	0.0461	-16.1648	-17.7151

We may see in Table 4 that results are very different when changing the method of calculating $\sigma^{cs}_{t-3,t-1}, s^{cs}_{t-3,t-1}, k^{cs}_{t-3,t-1}$. The only occasion when variables of V_3 do not keep their statistical significance, although with values larger than theoretical ones, is when IM is not used to obtain initial solutions. When using TR, V_3 has inconsistent values.

Table 5 presents results of V_4 relatively to observations from Table 2 and Table 4:

Table 5. Results of *lsqnonlin* for V_4 with TR, LM and IM

No. of. obs.	**TR**			**TR** with **IM**			**LM** with **IM**		
	$\sigma^{cs}_{x,t-1}$	$s^{cs}_{x,t-1}$	$k^{cs}_{x,t-1}$	$\sigma^{cs}_{x,t-1}$	$s^{cs}_{x,t-1}$	$k^{cs}_{x,t-1}$	$\sigma^{cs}_{x,t-1}$	$s^{cs}_{x,t-1}$	$k^{cs}_{x,t-1}$
3	0.0346	-68.2585	100	0.0346	-68.2585	100	0.0288	-167.0899	585.5107
4	0.0402	-30.5804	3	0.0402	-30.5804	3	0.0404	-30.1034	2.0520
5	0.0403	-30.4728	3	0.0403	-30.4728	3	0.0430	-20.2410	-15.7377
6	0.0403	-30.4615	3	0.0403	-30.4615	3	0.0423	-22.5352	-11.9254

In Table 5 lower bounds are (0, -100, 3) and upper bounds are (1, 0, 100) for $\sigma^{cs}_{x,t-1}, s^{cs}_{x,t-1}, k^{cs}_{x,t-1}$. These bounds are used for TR, which yield the same results using or not IM. More, for TR and TR using IM for initial solutions, the upper bound is reached once (i.e. first observation in Table 5) and lower bound is reached three times (i.e. last three observations in Table 5). This might raise questions about the final results if final results were not bounded. LM, using IM for initial solutions, proves that variables of V_4 might lose their statistical significance reaching values which are not theoretically consistent. Without using IM for initial solutions, LM cannot yield any result in most of the occasions.

We have seen (Table 4 and Table 5) that methods of determining the optimum solutions in a non-linear minimization may be difficult to be chosen. There is no guarantee that one or another method will yield a better if not an optimum result. The problem of choosing the best method is based on different attempts, intuition and graphics of one of the: call option price, underlying asset price or previous implied variables. Still, using IM to obtain initial solutions for any of the methods of non-linear minimization for more than one parameter, we obtain encouraging results for implied variables V_3 and V_4.

5 Final Results and Discussions

Both formula (1) and (2) are designed for a continuous time approach. We take this continuous time condition as being satisfied when a derivative instrument is subject to trading a few times each day from its birth to the expiration date. The call options analyzed in this paper have more than 1000 transactions during their entire life that can range from three to nine months and they are extracted from 35807 transactions (Appendix A.1). The transactions of 10 derivative instruments (Appendix A.2), taken into consideration in this paper, are classified by two keys: strike price *K* and expiration date *T* (i.e. Option ID) and sorting by day and time of trading. In Table 6 we present estimations on different options using the four implied variables V_1, V_2, V_3 and V_4:

Table 6. Estimations of call option price; *t* and *x* expressed as discrete observations. *N*: number of transactions observed for the entire life of the call option

No. of obs.	**Option ID**	**N**	**t**	**x**	**S**	**r**	**T-t**	**C**	CE^1_{t-1}	$CE^1_{x,t-1}$	CE^2_{t-3}	$CE^2_{x,t-1}$
1	2650_19970930	1672	570	560	2628.88	3.34	210	**125**	***124.0172***	123.0584	120.4606	128.8986
2	2650_19970930	1672	570	565	2628.88	3.34	210	**125**	124.0172	***124.3992***	120.4606	124.1808
3	2650_19970930	1672	570	566	2628.88	3.34	210	**125**	124.0172	***124.4366***	120.4606	123.0086
4	2800_19970930	2375	1200	1190	2792.65	3.41	132	**118**	***118.6034***	121.1909	***117.4134***	126.6547
5	2800_19970930	2375	1502	1485	2533.09	3.63	120	**32.5**	34.1319	38.5383	***33.0338***	35.3408
6	2950_19970930	2332	1962	1935	2808.17	3.41	29	**34**	***34.0998***	36.6215	0.0000	41.0334
7	2950_19970930	2332	1962	1956	2808.17	3.41	29	**34**	***34.0998***	35.4219	0.0000	***33.9085***

8	3250_19980331	1814	786	756	2732.54	3.66	152	**60**	***54.9276***	67.8185	76.1570	66.7305
9	3250_19980331	1814	786	776	2732.54	3.66	152	**60**	54.9276	***60.0936***	76.1570	60.7593
10	3250_19980331	1814	786	780	2732.54	3.66	152	**60**	54.9276	58.3534	76.1570	***60.0651***

There are four options, each with several estimations, presented in Table 6 and best estimation of the real traded call option price is highlighted (bold italic red color) for each observation. Before we continue with discussion on estimations, we also present in Table 7 the implied variables used to obtain data from Table 6, highlighting the implied variables that yield the best result:

Table 7. Implied variables that yield results from Table 6

No. of obs.	**Best method**	**V_1**	**V_2**	**V_3**			**V_4**		
		σ_{t-1}	$\sigma_{x,t-1}$	$\sigma^{cs}_{t-3,t-1}$	$s^{cs}_{t-3,t-1}$	$k^{cs}_{t-3,t-1}$	$\sigma^{cs}_{x,t-1}$	$s^{cs}_{x,t-1}$	$k^{cs}_{x,t-1}$
1	$CE^1_{t\text{-}1}$	***0.1375***	0.1363	-0.4003	6.5891	-2.4549	0.0135	-100.00	3.000
2	$CE^1_{x,t-1}$	0.1375	***0.1380***	-0.4003	6.5891	-2.4549	0.0129	-100.00	3.000
3	$CE^1_{x,t-1}$	0.1375	***0.1363***	-0.4003	6.5891	-2.4549	0.0128	-100.00	3.000
4	CE^2_{t-3}	0.1569	0.1608	***-1.5589***	***1.7959***	***1.6842***	0.0158	-100.00	3.000
5	CE^2_{t-3}	0.1798	0.1894	***0.4723***	***5.7970***	***26.7663***	0.0726	-46.4898	86.9866
6	$CE^1_{t\text{-}1}$	***0.2622***	0.2716	0.0136	0.0131	2.9995	0.0842	-0.0001	60.3895
7	$CE^2_{x,t\text{-}1}$	0.2622	0.2672	0.0136	0.0131	2.9995	***0.1007***	***-0.0001***	***38.2725***
8	$CE^1_{t\text{-}1}$	***0.2789***	0.3033	0.2771	-0.0722	23.3609	0.1224	-39.533	73.0404
9	$CE^1_{x,t-1}$	0.2789	***0.2889***	0.2771	-0.0722	23.3609	0.1510	-0.0001	15.1048
10	$CE^2_{x,t\text{-}1}$	0.2789	0.2855	0.2771	-0.0722	23.3609	***0.1469***	***-0.0001***	***15.3419***

Calculations made with V_i, i= 1, 4, proves consistent results for call option prices in Table 6 and intriguing implied values in Table 7.
Looking at call option that has the id 2650_19970930, first three observations in Table 6, we notice that a variations of x (time expressed as discrete observation to start calculations) for a fix t (given time expressed as given observation) leads to improved results. For the same observations, the implied variables that give the closest estimation to the real call option price are V_1 and V_2 (see Table 7).
For option with id 2800_19970930 for different moments of its life (i.e. different t) we obtain good estimations (see Table 6) with V_3 (see Table 7). But V_3 has in both cases values that lose their statistical significance. More, value of $\sigma^{cs}_{t-3,t-1}$ is negative for the first estimation (i.e. value of -1.5589) which is totally wrong from a theoretical point of view. Volatility must be non-negative, but is implied volatility the same theoretical measure of risk or uncertainty? We do not want to start a debate on acceptance of negative value for volatility, but we would like to point up that an economic and algebraic explanation would argue that negativity means a local tendency to contraction. We leave this debate for other researches; still, we would like to underline that not only implied volatility, skewness and kurtosis may have a reverse algebraic sign from an economic and statistic point of view, but also prices estimated with formula (2).
Option with id 2950_19970930 has outstanding estimations for a fix t with two different x and

different implied variables (see Table 7). More important is the fact that the same price yielded by V_3 are zero; it does not depend on x. There are occasions when prices estimated with V_3 and V_4 are even negative, which, on a superficial explanation, might be seen as a cost. Thus, in our case, for the last previous three transactions of call option at given moment t, the option becomes a burden for the investor. Still, the other methods of estimation prove that the price is fairly guessed (see Table 6).

The last option from Table 6, with id 3250_19980331, has two methods, based on V_2 and V_4, which estimate for a fix t and different x similar prices of call option. All implied variables V_i, $i = 1, 4$, keep their statistical significance for all three observations (see table 7).

We conclude our discussion on results and methods of estimation by stating that further research should investigate a pattern for choosing the best method of estimation.

6 Conclusions

This paper has the intention of proving that estimation on option markets, the most leveraged one, are at hand of traders if they look at the information that market retains through its transactions. Good estimations on call option pricing are possible if we treat each option as an autonomous asset that depends only on itself and its underlying evolution. We need correct numerical methods of calculating implied variables that yield the estimated price. We proved the superiority of bisection over Newton-Raphson method. We proposed an iterative method that finds a minimized implied volatility and initial solutions for three parameters non-linear minimization. This research on calculating implied variables has shown that some aberrations may occur. It is not easy to deal with this kind of errors and it is also difficult to find an economic explanation for them.

Although the values of the implied volatility, skewness and kurtosis are not in the range of a non-normal distribution with negative asymmetry and leptokurtic distribution, they represent, as implied parameters, new gauges of risk. The results obtained in estimating call option prices encourage us to believe that speculation is hardly profitable, taking into consideration only the information from previous transactions. Without information asymmetry, fair option price estimations are available for all traders, with the right method.

In a knowledge society, traders on derivative markets should be concerned with hedging, rather than speculating. It is true that liquidity is a very important aspect of economic world, but what are the medium and long term costs? We believe that traders should pay more attention and respect to the economic and non-economic world. Constraints on trading option must not come from state institution but from traders themselves.

This paper tries to emphasize the validity of at least one option pricing model. The author has proposed himself to verify from a computational and from a logical point of view two of the option pricing formulae. One strong assumption have yet been made, that options, and especially index-option, should be treated independently from the other options that have or do not have the same underlying. In this paper, the author assumes that a trader takes into consideration one kind of risk when one deals with one option. The liaison with other options, which mean another kind of investment and risk, it is totally explainable through the behavior of the underlying price. There is no point, in this article's author opinion, to group the options altogether and to determine implied parameters through a common behavior of all these derivatives. Traders know very well that the market is continuously transforming and they do not expect the same behavior from all options, even when they have the same class of maturity. We haven't discussed about the fact that options deep in or out of the money have their behavior exaggerated by Black-Scholes formula and we didn't even talk about the possibilities of arbitrage. We are assuming that the traded price is the real and objective price even when some aberrations might appear, like a decrease of the underlying price accompanied by an increase in the option call price.

Future researches should determine a priori the right method for call option pricing and to find a pattern based on underlying asset and option evolutions, and even on previous implied variables. Improvements of numerical methods may be based on algorithms that search deeply in the alternatives of multiple parameters non-linear minimization, even with the cost of time. The objective is to prove that estimations can be handy for all traders and, thus, we may prevent speculation on a high leverage market.

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Appendix

A. Transactions

A.1. CAC 40 transactions http://turcoane.com/ie/Resource1.txt

A.2. Transactions of ten call options http://turcoane.com/ie/Resource2.rar

B. Code source

B.1. Matlab implementation for bisection method http://turcoane.com/ie/Bisection.txt

B.2. Matlab implementation for Newton-Raphson method http://turcoane.com/ie/Raphson.txt

B.3. Matlab implementation for IM http://turcoane.com/ie/IM.txt

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