Hierarchical Planning Methodology for a Supply Chain Management

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Hierarchical production planning is a widely utilized methodology for real world capacitated production planning systems with the aim of establishing different decision–making levels of the planning issues on the time horizon considered. This paper presents a hierarchical approach proposed to a company that produces reusable shopping bags in Chile and Perú, to determine the optimal allocation of resources at the tactical level as well as over the most immediate planning horizon to meet customer demands for the next weeks. Starting from an aggregated production planning model, the aggregated decisions are disaggregated into refined decisions in two levels, using a couple of optimization models that impose appropriate constraints to keep coherence of the plan on the production system. The main features of the hierarchical solution approach are presented.

Keywords: *Supply Chain Management, Production Planning, Hierarchical Production Planning*

1 Introduction

The production management encompasses a large number of decisions such as how much to produce every week of the different finished items taking into account the customers' needs. Such decisions involve complex choices among a large number of alternatives imposed by financial, technological, and available resources, and marketing constraints that must be considered over a medium and short–term planning horizon. Hierarchical production planning (HPP) systems are developed with the aim of establishing different decision-making and information levels so that managers can concentrate on the most relevant aspects of the planning issues [9]. Many great contributions have been made using operations research to solve the production planning problems. In particular, different authors take the classical model proposed in [2] to test the robustness, coherence and feasibility of the disaggregation process; see e.g. [1], [3], [5]. The most important task consists in providing, with HPP, a robust, stable and feasible plan taking into account capacity allocation and priority management at the master production scheduling level. In a typical hierarchical production planning model, the objective is mainly to decompose a large and complex planning problem into less complex planning subproblems resulting in consistent aggregate and master production schedules. In the production system at hand, the fact that demands of semi-finished products are relatively stable suggests that, even though demands of the finished products are random, planning at the level of semi-finished products may have some stabilizing effect on the aggregate production planning of the whole system.

On the other hand, an optimal production plan must consider the whole supply chain in its three distinct stages: manufacturer/supplier of product-specific materials (parts), producer where finished products are assembled according to customer orders, and a set of customers who generate final demand for the products. The supply chain problem using a methodology HPP was originally study in [6] for production and distribution decisions, in this paper we consider the supply chain of FER CREACIONES Ltd., a firm that produce reusable shopping bags, and propose several optimization models based on the HPP methodology with the purpose of establishing an optimal production policy to meet customer demands for the next weeks, starting with an aggregate production planning problem. To discuss the main features of this contribution, in the next section the different optimization models in the context of HPP structure are described. Thus, an industrial case and the preliminary results obtained using the adopted methodology are summarized. Finally, some conclusions and future work are presented.

2 Optimization models for the HPP strategy

At present it is essential to use optimization models for solving complex problems in production and distribution logistics. In this section we propose three different models in the framework of the adopted methodology which, starting with a tactical production planning model, then introduce two optimization models that maintain the coherence between aggregated tactical decisions and those adopted per week.

A tactical production planning problem is one of the main problems concerning medium–term decisions in operations management, and its use in the context of a particular SC allows a well integrated coordination with suppliers and sales, see, e.g., [4], [7], [8]. The proposed model for this problem provides an optimal production policy that minimizes total production costs in order to meet forecast demands for the different product families.

Assuming a finite and discrete planning horizon, the following notation is used to present a linear optimization model for this problem. Let T be the total number of periods (months). Let F be the set of finished family products, M the set of raw materials, and P the set of suppliers. The main parameters of the model are the demand d_{it} for family product i in period t and different variable cost for transportation, production, inventory and labor force. On the other hand, the decision variables of the models include the total amounts of units QT_{mpt} of raw material m bought from supplier p in period t, the total amounts of units Q_{mpit} of raw material m bought from supplier p to produce family product i in period t, the number of units X_{it} to be produced of family product i in period t, the number of inventory units I_{it} of family i in period t, the number of unmet demand units U_{it} of family i in period t, the number of hours in regular time R_t in period t, and the number of overtime hours O_t used during period t. Then the tactical model can be formulated as follows:

$$Min\ \sum_{m\epsilon M}\sum_{p\epsilon P}\sum_{t=1}^{T} q_{mpt}QT_{mpt} + \sum_{i\epsilon F}\sum_{t=1}^{T}[c_{it}X_{it} + h_{it}I_{it} + f_{it}U_{it}] + \sum_{t=1}^{T}[r_tR_t + o_tO_t]$$

s.t.

$X_{it} + I_{it-1} - I_{it} + U_{it} = d_{it}$ for $i \epsilon F$, $t = 1,\dots, T$. (2)

$\sum_{i\epsilon F} a_iX_{it} = R_t + O_t$ for $t = 1,\dots, T$. (3)

$\sum_{p\epsilon P} Q_{mpit} = n_{mi}X_{it}$ for $m \epsilon M, i \epsilon F, t = 1,\dots, T$. (4)

$\sum_{i\epsilon F} Q_{mpit} = QT_{mpt}$ for $m \epsilon M, p \epsilon P, t = 1,\dots, T$ (5)

$R_t \le rm_t$ for $t = 1,\dots, T$ (6)

$O_t \le om_t$ for $t = 1,\dots, T$ (7)

$U_{it} \le \sigma_1 d_{it}$ for $i \epsilon F, t = 1,\dots, T$ (8)

$X_{it} \ge 0, I_{it} \ge 0, U_{it} \ge 0, R_t \ge 0, O_t \ge 0$ for $i \epsilon F, t = 1,\dots, T$ (9)

The objective function in (1) minimizes the total cost, defined by transportation, production, inventory, shortage and labor resources costs. Equations (2) establish that the required demand for each family product must be satisfied using production and inventory, but allowing shortage eventuality, where I_{0i} represents the initial inventory of family product i. Constraints (3) correspond to capacity constraints where the total time requirement for producing the different items, at a rate of a_i hours per unit of family product i, must be equal to the time availability in regular and overtime hours. Constraints (4) and (5) allow to compute the total amounts of units of raw material bought to the different suppliers, where n_{mi} is the number of units of raw material m necessary to produce one unit of family product i. Constraints (6), (7) and

(8) impose an upper bound in the utilization of regular and overtime hours and the maximum number of shortage units, respectively. Finally, (9) are the non-negativity constraints.
Next, the optimal decision variables obtained in the aggregated plan (1)–(9) are disaggregated by family in production plans for every item $j \in J(i)$ belonging to family *i*, now considering the minimization of the available inventory levels of the family, because of their financial impact. The decision variables in the family disaggregation model are the number of units Y_{jt} of item *j* to be produced in period *t*, the available inventory Iy_{jt} of item *j* in period *t* and the unmet demand V_{jt} of item *j* at each month. The model considers the demand requirements for each item and coherence between the family decisions in the aggregate plan according to the following model, solved separately for each family $i \in F$:

$$Min \sum_{j\epsilon J(i)} \sum_{t=1}^{T} \left[\frac{X_{it}+I_{it-1}}{\sum_{j\epsilon J(i)} d_{jt}} - \frac{Y_{jt}+Iy_{jt-1}}{d_{jt}}\right]^2 \quad (10)$$

$s.t.\quad Y_{jt} + I_{jt-1} - Iy_{jt} + V_{jt} = d_{jt}$ for $j \epsilon J(i)$, $t = 1,\dots, T$. (11)

$\sum_{j\epsilon J(i)} Y_{jt} = X_{it}^{*}$ for $t = 1,\dots, T$. (12)

$\sum_{j\epsilon J(i)} Iy_{jt} = I_{it}^{*}$ for $t = 1,\dots, T$. (13)

$\sum_{j\epsilon J(i)} W_{jt} = U_{it}^{*}$ for $t = 1,\dots, T$. (14)

$V_{jt} \le \sigma_2 d_{jt}$ for $j \epsilon J(i)$, $t = 1,\dots, T$. (15)

$Y_{jt} \ge 0, Iy_{jt} \ge 0, V_{jt} \ge 0$ for $j \epsilon J(i)$, $t = 1,\dots, T$. (16)

In this model, the quadratic objective function in (10) minimizes the total variation between the available items for each product with respect to the available units in the corresponding product family. Constraints (11) state that the required demand for each item must be satisfied using production and inventory, but allowing the eventuality of unmet demand units. Constraints (12), (13) and (14) impose that the total number of produced items, inventory units, and allowable unmet demand units must be equal to the family level of production, inventory, and unmet demand level according to the aggregated plan (1)-(9), respectively. Constraints (15) impose again an upper bound in the number of shortage units for each product, and constraints (16) impose the non-negativity on the different decision variables.
Finally, at the second level of disaggregation, we propose a linear optimization model to obtain a weekly production for each item, that we solve separately for each product *j* of the different families and clients $c \in C(j)$ that demand product *j*, considering as input data the optimal solutions of monthly production reached in model (10)-(16). The optimal solution for product *j* is taken according to the following linear optimization model:

$$Min \sum_{t=1}^{T^*} \sum_{k=1}^{4} \sum_{c\epsilon C(j)} f_{ktc} W_{ktc} \quad (17)$$

s.t. $\mathrm{Z_{ktc} + Iz_{kt-1c} - Iz_{ktc} + W_{ktc} = d_{ktc}}$ for $t = 1,\dots, T^*$, $k=1, 2, 3, 4$, $c \epsilon C(j)$. (18)

$\sum_{c\epsilon C(j)} \sum_{k=1}^{4} Z_{ktc} = Y_{jt}^{*}$ for $t = 1,\dots, T^*$. (19)

$\sum_{c\epsilon C(j)} \sum_{k=1}^{4} \mathrm{Iz_{ktc}} = \mathrm{Iy_{jt}^{*}}$ for $t = 1,\dots, T^*$.(20)

$\sum_{c\epsilon C(j)} \sum_{k=1}^{4} W_{ktc} = W_{it}^{*}$ for $t = 1,\dots, T^*$.(21)

$W_{ktc} \le \sigma_3 d_{ktc}$ for $t = 1,\dots, T^*$, $k = 1,2,3,4$, $c \epsilon C(j)$. (22)

$Z_{ktc} \ge 0, Iz_{ktc} \ge 0, W_{ktc} \ge 0$ for $t = 1,\dots, T^*$, $k = 1,2,3,4$, $c \epsilon C(j)$. (23)

Model (17)-(23) includes decisions per week over the most immediate planning months $t = 1,..,T^*$, considering as decisions variables the number of units Z_{ktc} to be produced in period *k* of month *t* for clients *c*, the inventory level Iz_{ktc} in period *k* of month *t* for clients *c*, and the unmet demand W_{ktc} of products at each period for clients c. The objective function in (17) minimizes the sum of the unmet demand units according to the relative importance of the different orders of items for each client. Constraints (18) establish the demand requirement during each week, according to the given demand d_{ktc} of client *c*, allowing shortage units. Constraints (19), (20) and (21) impose that the total number of produced items, inventory units and allowable unmet demands units must be equal to the optimal decision value of production, inventory and unmet demand in the first level of disaggregation, respectively. Constraints (22) impose an upper bound in the number of shortage units in each week, (23) are the non-negativity constraints for all the decision variables.

3 Case study

The proposed methodology was preliminary tested using different instances considering a production planning problem at FER CREATIONS Ltd., a leader company in the Chilean market of reusable shopping bags that is also starting to produce and market its products in Perú. Under the trade name BolsasReutilizables.cl, this company is manufacturing and selling reusable bags for retail, based on a mixed business and operation model focused on low costs and good service, delivering attractive bag models and designs in agreement with their customers' needs.

With respect to the propose methodology, the resulting models were represented on the algebraic modeling language AMPL, with CPLEX 11.2 as a linear and quadratic solver. The tactical production planning model (1)-(9) considers a planning horizon of one year divided into monthly periods, 6 family products and 8 main raw materials, which has 1584 continuous decision variables and around 500 constraints. Future demands were estimated from historical data by fitting appropriate curves using the FindGraph software. The results show that the model makes use of the different flexibilities delivered for planning production, i.e., using inventories, overtime, and unfulfilled demand units in some cases. This model was also analyzed considering different scenarios associated with changes in the demand and the availability of raw materials by the suppliers, which show its goodness and the stability of the resulting solutions. Then, the first three months define the frozen horizon considered in the disaggregation models (10)-(16) and (17)-(23). Once the aggregated plan has been determined, a first disaggregation model is then solved that provides an optimal production policy for each family and for the 15 products analyzed. The model (10)-(16) is quadratic and could be solved with no difficulty for each of the families, taking the options provided by the use of inventory and shortage, respecting the decisions of the higher hierarchical level. Finally, the second disaggregation level is solved separately for each product, considering a total of twelve periods in the first 3 months of the initial planning horizon. The production volumes determined in each period show the stability of the adopted solutions and allow filling the needs according to the client's most immediate demands, aiming at the required inventory levels to meet the future demand that has been projected beyond the first three months.

4 Conclusions and extensions

We propose a methodology based on the hierarchical production planning approach that could be applied to a manufacturing supply chain in order to disaggregate a family production plan into decisions per week for the different finished products to be produced in the short-term planning horizon. The approach is shown to be suitable to deal with a real manufacturing system. The application of the proposed hierarchical system succeeds in decomposing adequately the planning problem through different models that allow decreasing the number of decision variables and restrictions involved in the problem as a whole and disaggregating properly the relations between the organization and its supply chain to make medium and short-term decisions as appropriate. Finally, we leave open the extension of this methodology to an alternative stochastic formulation with the inclusion of different scenarios capturing part of the uncertainty of the system related to future demands, costs and availability of raw materials, taking into account that such extensions would also require the development of suitable numerical strategies to solve them.

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